WEEKLY TEST OYJ TEST - 28 R & B
SOLUTION Date 17-11-2019

[PHYSICS]

1. **(a)** It is given that energy remains the same.

Hence, $E_A = E_B$

Energy $\propto a^2 n^2 \Rightarrow \frac{a_B}{a_A} = \frac{n_A}{n_B}$ ($\because$ energy is same)

$\therefore \left(\frac{a_A}{a_B}\right)^2 = \left(\frac{n_B}{n_A}\right)^2$

Given, $n_A = n, n_B = \frac{n}{8}$

$\therefore \frac{a_A}{a_B} = \frac{n/8}{n} = \frac{1}{8} \Rightarrow a_B = 8a_A = 8a$

2. **(a)** Time required for a point to move from maximum displacement to zero displacement is

$t = \frac{T}{4} = \frac{1}{4n}$

$\Rightarrow n = \frac{1}{4t} = \frac{1}{4 \times 0.170} = 1.47$ Hz

3. **(a)** The apparent change in the frequency of the source due to relative motion between source and observer is known as Doppler's effect. The perceived frequency (n') when listener is static and source is moving away is given by

$$n' = n\left(\frac{v}{v+v_s}\right)$$

where n is frequency of source, v is velocity of sound and v_s is velocity of source.

Putting $v = 330$ m/s, $v_s = 30$ m/s, $n = 800$ Hz.

$$n' = 800 \times \left(\frac{330}{330+30}\right)$$

$$n' = 733.3 \text{ Hz}$$

In the limit when speed of source and observer is much lesser than that of sound v_1, the change in frequency becomes independent of the fact whether the source is moved or the detector.

4. **(b)** Since the point $x = 0$ is a node and reflection is taking place from point $x = 0$. This means that reflection must be taking place from the fixed end and hence the reflected ray must suffer an additional phase change of π or a path change of $\lambda/2$

So, if $y_{\text{incident}} = a\cos(kx - \omega t)$

$$\Rightarrow \quad y_{\text{reflected}} = a\cos(-kx - \omega t + \pi)$$

$$= -a\cos(\omega t + kx)$$

5. **(d)** Points B and F are in same phase as they are λ distance apart.

6. **(c)** Critical hearing frequency for a person is 20,000 Hz.

If a closed pipe vibration in N^{th} mode then frequency of vibration

$$n = \frac{(2N-1)v}{4l} = (2N-1)n_1$$

(where n_1 = fundamental frequency of vibration)

Hence $20{,}000 = (2N - 1) \times 1500 \Rightarrow \text{N} = 7.1 \approx 7$

Maximum possible harmonics obtained are

1,3,5,7,9,11,13

Hence, man can hear up to 13$^{\text{th}}$ harmonic

$= 7 - 1 = 6$

So, number of overtones heard = 6

7. **(a)** In first overtone mode, $l = \frac{3\lambda}{4}$

$\therefore \quad \frac{\lambda}{4} = \frac{l}{3} = \frac{1.2}{3} = 0.4 \text{ m}$

Pressure variation will be maximum at displacement nodes, i.e., at 0.4 m from the open end.

8. **(c)** The frequency of A, $n_A = n + \frac{2}{100}n$

and the frequency of B, $n_B = n - \frac{3}{100}n$

According to question, $n_A - n_B = 6$

$\therefore \quad \left(n + \frac{2}{100}n\right) - \left(n - \frac{3}{100}n\right) = 6$

or $\quad \frac{5}{100}n = 6 \quad \Rightarrow \quad n = \frac{600}{5} = 120 \text{ Hz}$

The frequency of A

$$n_A = \left(n + \frac{2}{100}n\right) = 120 + \frac{2}{100} \times 120$$

$$= 122.4 \text{ Hz}$$

9. **(c)** Fundamental frequency of closed pipe

$$n = \frac{v}{4l} = 220 \text{ Hz} \Rightarrow v = 220 \times 4l$$

If 1/4 of the pipe is filled with water then remaining length of air column is $\frac{3l}{4}$

Now fundamental frequency $= \frac{v}{4\left(\frac{3l}{4}\right)} = \frac{v}{3l}$ and

First overtone = 3 × fundamental frequency

$$= \frac{3v}{3l} = \frac{v}{l} = \frac{220 \times 4l}{l} = 880 \text{ Hz}$$

10. **(c)** $f \propto \sqrt{T}$

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow \frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta T}{T}$$

$\Rightarrow \quad \Delta f = \frac{202}{2} \times \frac{1}{101} = 1$

11. **(d)** $\frac{\nu_1}{\nu_2}=\frac{28}{27}$

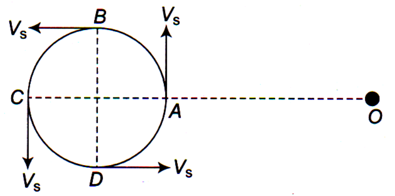
$$\nu_1-\nu_2=3 \text{ or } \frac{28}{27}\nu_2-\nu_2=3$$

$$\nu_2=27\times 3\text{ Hz}=81\text{ Hz}$$

or $$\nu_1=\nu_2+3=(81+3)\text{Hz}$$

or $$\nu_1 = 84\text{ Hz}$$

12. **(d)** Frequency heard by the observed will be maximum when the source is in the position *D*. In this case, source will be approaching towards the stationary observer, almost along the line of sight (as observer is stationed at a larger distance).

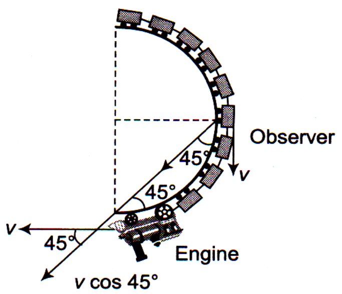


Similarly, frequency heard by the observer will be minimum when the source reaches at the position *B*. Now, the source will be moving away from the observer.

$$n_{\text{min.}}=\frac{v}{v+v_s}\times n=\frac{330}{330+1.5\times 20}\times 440$$

$$=\frac{330\times 440}{360}=403.3\text{ Hz}$$

13. **(c)** The situation is shown in the fig. Both the source (engine) and the observer (Person in the middle of the train) have the same speed, but their direction of motion is right angles to each other. The component of velocity of observer towards source is v cos 45° and that of source along the time joining the observer and source is also v cos 45°. There is number relative motion between them, so there is no change in frequency heard. So frequency heard is 200 Hz.

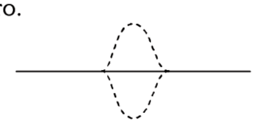


14. (b) Equation of A, B, C and D are
$y_A = A\sin\omega t$, $y_B = A\sin(\omega t + \pi/2)$
$y_C = A\sin(\omega t - \pi/2)$, $y_D = A\sin(\omega t - \pi)$
It is clear that wave C lags behind by a phase angle of $\pi/2$ and the wave B is ahead by a phase angle at $\pi/2$.

15. (c) The particle velocity is maximum at B and is given by
$\frac{dy}{dt} = (v_p)_{\max} = \omega A$
Also wave velocity is $\frac{dx}{dt} = v = \frac{\omega}{k}$
So slope $\frac{dy}{dx} = \frac{(v_p)_{\max}}{v} = kA$

16. (c) We know frequency $n = \frac{p}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} \Rightarrow n \propto \frac{1}{\sqrt{\rho}}$
i.e., graph between n and $\sqrt{\rho}$ will be hyperbola.

17. (c) After two seconds each wave travel a distance of 2.5 × 2 = 5 *cm i.e.* the two pulses will meet in mutually opposite phase and hence the amplitude of resultant will be zero.



18. (c) $n_Q = 341 \pm 3 = 344Hz$ or $338Hz$
on waxing Q, the number of beats decreases hence $n_Q = 344Hz$

19.

20. (d) Intensity $\propto a^2\omega^2$
here $\frac{a_A}{a_B} = \frac{2}{1}$ and $\frac{\omega_A}{\omega_B} = \frac{1}{2} \Rightarrow \frac{I_A}{I_B} = \left(\frac{2}{1}\right)^2 \times \left(\frac{1}{2}\right)^2 = \frac{1}{1}$

[MATHEMATICS]

41. (a) $P(X + Y = 3) = P(X = 0, Y = 3) + P(X = 1, Y = 2) + P(X = 2, Y = 1) + P(X = 3, Y = 0)$
$= P(X = 0)\, P(Y = 3) + P(X = 1)\, P(Y = 2) + P(X = 2)\, P(Y = 1) + P(X = 3)\, P(Y = 0)$ (X and Y are independent)

$= {}^5C_0\,(1/2)^5\;{}^7C_3\,(1/2)^7 + {}^5C_1\,(1/2)^5\;{}^7C_2\,(1/2)^7 + {}^5C_2\,(1/2)^5;\;{}^7C_1\,(1/2)^7 + {}^5C_3\,(1/2)^5\,{}^7C_0\,(1/2)^7$
$= (1/2)^{12}\{(1)(35) + (5)(21) + (10)(7) + (10)(1)\}$
$= 220/2^{12} = 55/1024$

42. (a) Let x_1 x_2 x_3 x_4 x_5 x_6 be the number then each of x_1, x_2, x_3, x_4, x_5 has 9, 10, 10, 10, 10 choices respectively. Now summing these five digits the sum is either odd or even. If it is odd, take $x_6 \in \{1, 3, 5, 7, 9\}$ and if it is even, take $x_6 \in \{0, 2, 4, 6, 8\}$ so that the sum of six digits becomes even. Thus number of desired type of numbers = $9 \times 10^4 \times 5$.

Thus $P(E) = \dfrac{9\times10^4\times5}{9\times10^5} = \dfrac{1}{2}$

43. (b) If first throw is four then sum of numbers appearing on last two throws must be equal to eleven. That means last two throws are (6, 5) or (5, 6)
Now there are 10 ways to get the sum as 15.
i.e., {(4, 5, 6), (4, 6, 5),..., (3, 6, 6)}

$\Rightarrow$ Required Probability = $\dfrac{2}{10} = \dfrac{1}{5}$

44. (b) Third six occurs on 8th trial. It means that in first 7 trials we must have exactly 2 sixes and 8th trial must result in a six

$\Rightarrow$ Required probability = ${}^7C_2$ $(1/6)^2$. $(5/6)^5$. $(1/6)$

$= \dfrac{{}^7C_2\, 5^5}{6^8}$

45. (c) $A_1, A_2, A_3, \ldots, A_n$ are n independent events .

Now $P(A_i) = \dfrac{1}{i+1}$

P (None of the events will occur)

$= P\left(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \ldots\ldots\ldots \cap \bar{A}_n\right) = P(\bar{A}_1)P(\bar{A}_2)P(\bar{A}_3)\ldots\ldots(\bar{A}_n)$

$= \left(1-\dfrac{1}{2}\right)\left(1-\dfrac{1}{3}\right)\left(1-\dfrac{1}{4}\right)\ldots\ldots\left(1-\dfrac{1}{n+1}\right)$

$= \dfrac{1}{n+1}\quad \dfrac{1}{2}.\dfrac{2}{3}.\dfrac{3}{4}\ldots\ldots\dfrac{n}{n+1} = \dfrac{1}{n+1}$

$= \dfrac{1}{2}\times\dfrac{2}{3}\times\dfrac{3}{4}\times\ldots\times\dfrac{n}{n+1} = \dfrac{1}{n+1}$

46. **(c)** The number of determinants formed = 16. Observe that the determinant is non-zero when exactly once (–1) appears

as shown $\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2 \Rightarrow$ 4 ways

Similarly the determinant is non-zero when (–1) is used exactly three times as shown $\begin{vmatrix} -1 & -1 \\ -1 & 1 \end{vmatrix} = -2 \Rightarrow$ 4 ways

So non-zero determinant can be obtained in 8 ways. Similarly determinant will be zero in 8 determinants

$\Rightarrow P(E) = 1/2$

47. **(c)** $P(A^C) = 0.3 \Rightarrow p(A) = 0.7$
$P(B) = 0.4$
$P(A \cap B^c) = 0.5 \Rightarrow P(A) - P(A \cap B) = 0.5$
So $P(A \cap B) = 0.2$
Further $P(A \cup B^C) = P(B^C) + P(A \cap B)$
{Since (B^C) and $(A \cap B)$ are mutually exclusive or say disjoint events}
So $P(A \cup B^C) = 0.8$
Also $P(B \cap (A \cup B^C)) = P(A \cap B) = 0.2$
Hence $P\left(\frac{B}{A \cup B^C}\right) = \frac{0.2}{0.8} = \frac{1}{4}$

48. **(b)** $P(A) = 0.2 \Rightarrow P(A') = 0.8$ and $P(B) = 0.5$; $P(A' \cap B) = P(B) - P(A \cap B) \leq P(B)$
$\therefore$ Maximum value of $P(A' \cap B) = P(B) = 0.5$

49. (b) $P(A_1 \cup A_2) = 1 - P(A_1^c).P(A_2^c)$
$= 1 - \{1 - P(A_1)\}\{1 - P(A_2)\} = 1 - 1 + P(A_1) + P(A_2) - P(A_1)P(A_2)$
$\Rightarrow P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1).P(A_2)$
But $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$
$\Rightarrow P(A_1 \cap A_2) = P(A_1)P(A_2)$ $\therefore$ independent events

50. **(c)** Let the numbers be x, y.
Given the sum of two positive real numbers $= x + y = 2a$.
$\therefore$ Maximum product $= a^2$
Permissible product $\in \left[\frac{3a^2}{4}, a^2\right]$
Now $\frac{3a^2}{4} = a^2 - \frac{a^2}{4} = \left(a + \frac{a}{2}\right)\left(a - \frac{a}{2}\right)$
$\Rightarrow x, y, \in \left[\frac{a}{2}, \frac{3a}{2}\right]$; The required probability $= \frac{a^2}{4a^2} = \frac{1}{4}$

51. (d) Let A denotes the event that the student is selected in IIT entrance test and B denotes the event that he is selected in Roorkee entrance test. Then
$P(A) = 0.2$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$.
Required probability $= P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$
$= 1 - (P(A) + P(B) - P(A \cap B)) = 1 - (0.2 + 0.5 - 0.3) = 0.6$.

52. (a) Here the least number of draws to obtain 2 aces are 2 and the maximum number is 50 thus n can take value from 2 to 50.
Since we have to make n draws for getting two aces, in $(n-1)$ draws, we get any one of the 4 aces and in the n^{th} draw we get one ace. Hence the required probability $= \frac{{}^4C_1 \times {}^{48}C_{n-2}}{{}^{52}C_{n-1}} \times \frac{3}{52-(n-1)}$
$= \frac{4 \times (48)!}{(n-2)!(48-n+2)!} \times \frac{(n-1)!(52-n+1)!}{(52)!} \times \frac{3}{52-n+1}$
$= \frac{(n-1)(52-n)(51-n)}{50 \times 49 \times 17 \times 13}$ (on simplification).

53. (c) $n(S) = 6 \times 6 \times 6$

$n(E) =$ The number of solutions of $x + y + z = 7$,

where $1 \le x \le 6,\ 1 \le y \le 6,\ 1 \le z \le 6$

$=$ coefficient of x^7 in $(x + x^2 + ... + x^6)^3$

$=$ coefficient of x^4 in $(1 + x + ... + x^5)^3$

$=$ coefficient of x^4 in $\left(\frac{1-x^6}{1-x}\right)^3$

$=$ coefficient of x^4 in $(1 - 3.x^6 + 3.x^{12} - x^{18})(1-x)^{-3}$

$=$ coefficient of x^4 in $(1 - 3x^6 + 3x^{12} - x^{18})$

$({}^2C_0 + {}^3C_1 x + {}^4C_2 x^2 + {}^5C_3 x^3 + {}^6C_4 x^4 + ...)$

$= {}^6C_4 = \frac{6\,!}{4\,!.2\,!} = \frac{6 \times 5}{2} = 15$

$\therefore\ p(E) = \frac{n(E)}{n(S)} = \frac{15}{6 \times 6 \times 6} = \frac{5}{72}$.

54. (d) Let p be the probability of the other event, then the probability of the first event is $\frac{2}{3}p$. Since two events are

toally exclusive, we have $p + \left(\frac{2}{3}\right)p = 1 \Rightarrow p = \frac{3}{5}$

Hence odds in favour of the other are $3 : 5 - 3$, *i.e.* $3 : 2$.

55. (c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cap B)$

$\{\because\ P(A \cap B) = P(A \cup B)\}$

$\Rightarrow 2P(A \cap B) = P(A) + P(B)$

$\Rightarrow 2P(A).\frac{P(A \cap B)}{P(A)} = P(A) + P(B)$

$\Rightarrow 2P(A).P\left(\frac{B}{A}\right) = P(A) + P(B)$.

56. (b) $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

Since A and B are mutually exclusive, so

$P(A \cup B) = P(A) + P(B)$

Hence required probability $= 1 - (0.5 + 0.3) = 0.2$.

57. (a) Let the events are

$R_1 = A$ red ball is drawn from urn A and placed in B

$B_1 = A$ black ball is drawn from urn A and placed in B

$R_2 = A$ red ball is drawn from urn B and placed in A

$B_2 = A$ black ball is drawn from urn B and placed in A

$R = A$ red ball is drawn in the second attempt from A

Then the required probability

$= P(R_1R_2R) + (R_1B_2R) + P(B_1R_2R) + P(B_1B_2R)$

$= P(R_1)P(R_2)P(R) + P(R_1)P(B_2)P(R) + P(B_1)P(R_2)P(R) +$
$P(B_1)P(B_2)P(R)$

$= \frac{6}{10}\times\frac{5}{11}\times\frac{6}{10}+\frac{6}{10}\times\frac{6}{11}\times\frac{5}{10}+\frac{4}{10}\times\frac{4}{11}\times\frac{7}{10}+\frac{4}{10}\times\frac{7}{11}\times\frac{6}{10}$

$= \frac{32}{55}$.

58. (d) We have $P(A \cup B) \geq \max.\ \{P(A), P(B) = \frac{2}{3}$

$P(A \cap B) \leq \min.\ \{P(A), P(B)\} = \frac{1}{2}$

and $P(A \cap B) = P(A) + P(B) - P(A \cup B) \geq P(A) - P(B) - 1 = \frac{1}{6}$

$\Rightarrow \frac{1}{6} \leq P(A \cap B) \leq \frac{1}{2}$

$P\,(A' \cap B) = P(B) - P(A \cap B)$

Hence $\frac{2}{3} - \frac{1}{2} \leq P(A' \cap B) \leq \frac{2}{3} - \frac{1}{6}$

$\Rightarrow \frac{1}{6} \leq P(A' \cap B) \leq \frac{1}{2}$.

59. (d) Let a white ball be transferred from the first bag to the second. The Probability of selecting a white ball from the first bag $= \frac{5}{9}$.

Now the second bag has 8 white and 9 black. The probability of selecting white ball from the second bag $= \frac{8}{17}$.

Hence required probability $= \frac{5}{9}\times\frac{8}{17} = \frac{40}{153}$

If a black ball be transferred from the first bag to the second, then the probability $= \frac{4}{9}\times\frac{7}{17} = \frac{28}{153}$

Therefore required probability $= \frac{40}{153} + \frac{28}{153} = \frac{4}{9}$.

60. (d) Let E be the event that a new product is introduced.

Then $P(A) = 0.5$, $P(B) = 0.3$, $P(C) = 0.2$

and $P(E/A) = 0.7,\ P(E/B) = 0.6,\ P(E/C) = 0.5.$

$\because$ A, B and C are mutually exclusive and exhaustive events.

$P(E) = P(A).P(E/A) + P(B).P(E/B) + P(C).P(E/C)$

$= 0.5\times0.7 + 0.3\times0.6 + 0.2\times0.5$

$= 0.35 + 0.18 + 0.10 = 0.63.$